Geodesy 1 (GED203)

Lecture No: 2

GEOMETRIC PROPERTIES OF THE ELLIPSOID

Reda FEKRY

reda.abdelkawy@feng.bu.edu.eg

Review of Previous Lecture

Definition of geodesy

Objective of Geodesy

Branches of Geodesy

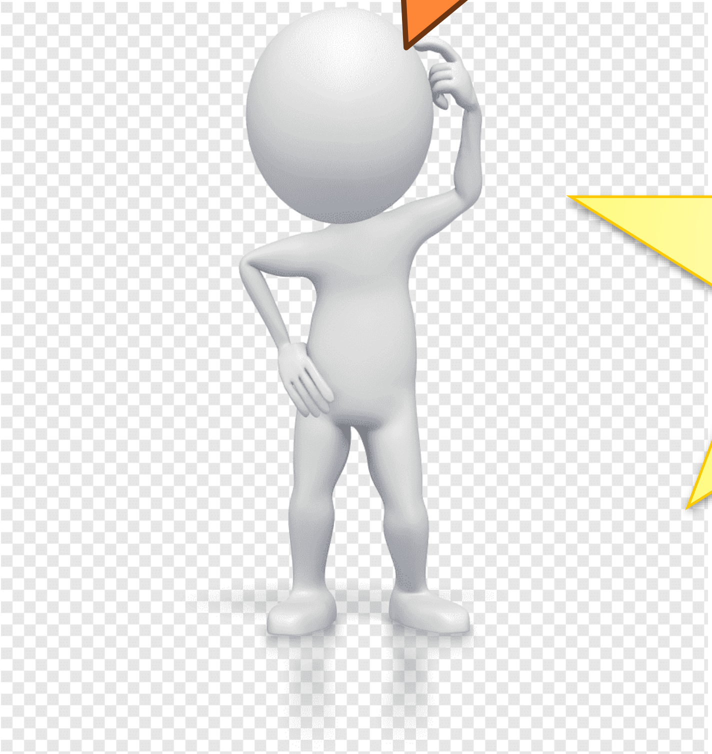
History of Geodesy

Shape and Size of The Earth

Latitude and Longitude

Geoid

Where are we till now?



please keep up

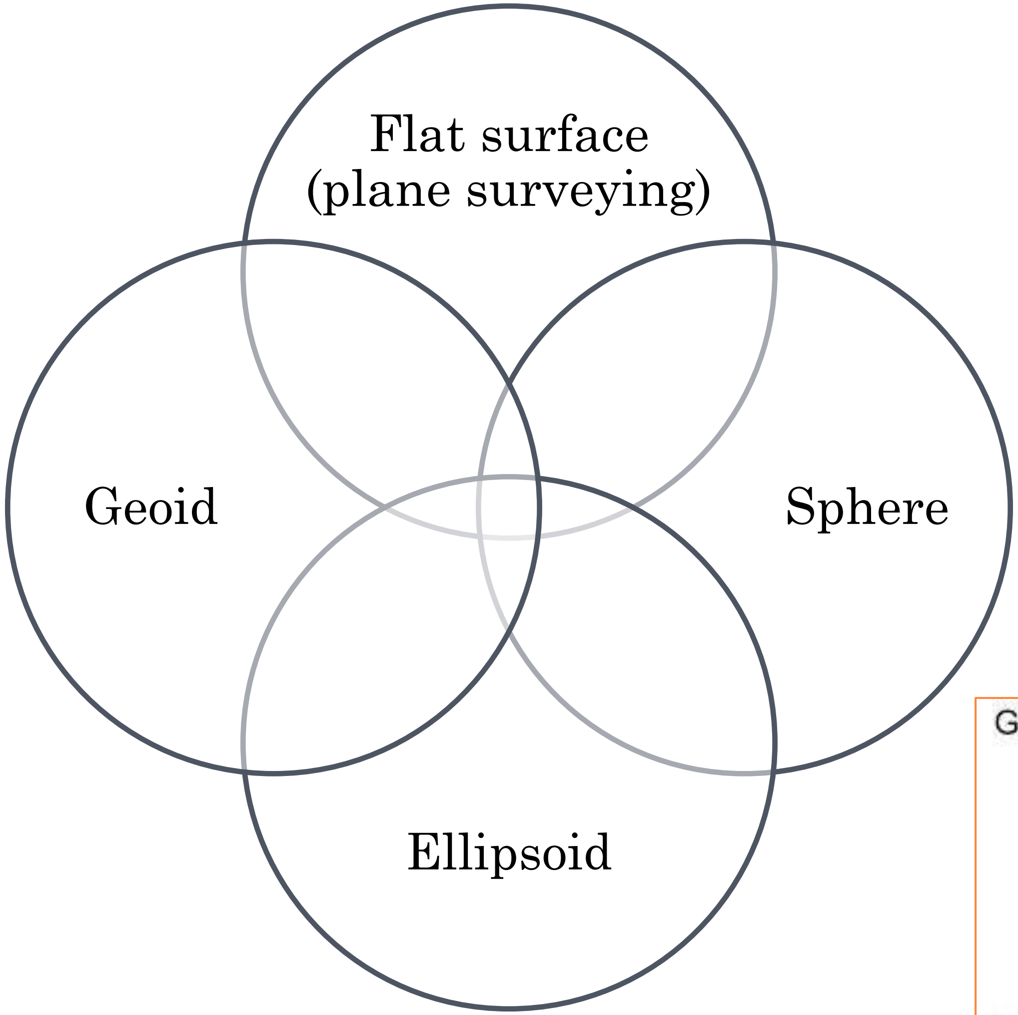
Expected Learning Outcomes

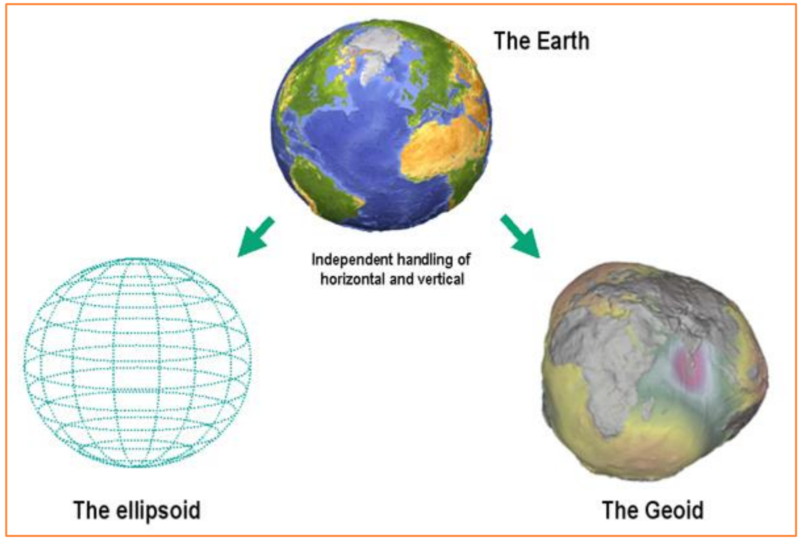
- Understanding the concept of an ellipsoid as a geometric shape that approximates the Earth's shape.
- Knowledge of linear and angular parameters of the ellipsoid.
- Understanding the ellipsoid as a reference frame.
- Familiarity with geodetic (ellipsoidal) coordinates.
- Knowledge of various types of latitudes, including geodetic latitude, geocentric latitude, and reduced latitude, and understand their significance in geodetic calculations.
- learning about the connections and differences between geodetic, geocentric, and reduced latitudes and how they relate to each other mathematically.
- Understanding the relationship between topography, ellipsoid, and geoid.
- Understanding the concept of selecting the best-fitting ellipsoid that closely approximates the Earth's shape for specific geodetic applications.

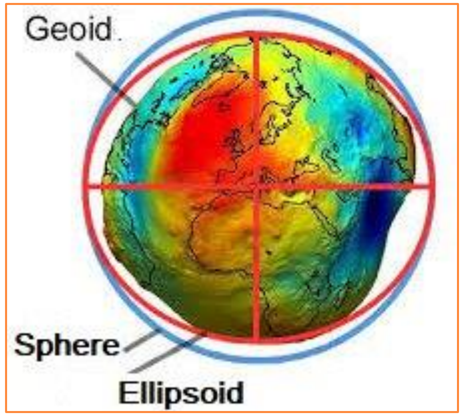
OVERVIEW OF TODAY'S LECTURE

- SHAPES THAT APPROXIMATES THE EARTH'S SHAPE →
- ELLIPSOID GEOMETRY →
- LINEAR AND ANGULAR PARAMETERS OF ELLIPSOID →
- ELLIPSOID AS REFERENCE FRAME →
- GEODETIC (ELLIPSOIDAL) COORDINATES →
- DIFFERENT TYPES OF LATITUDES →
- RELATION BETWEEN TYPES OF LATITUDES →
- RELATION BETWEEN TOPOGRAPHY, ELLIPSOID, AN GEOID →
- BEST FITTING ELLIPSOID →
- SUMMARY

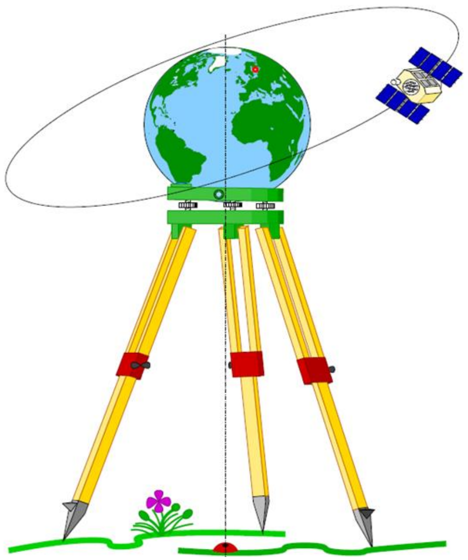
SHAPES THAT APPROXIMATE THE EARTH'S SHAPE







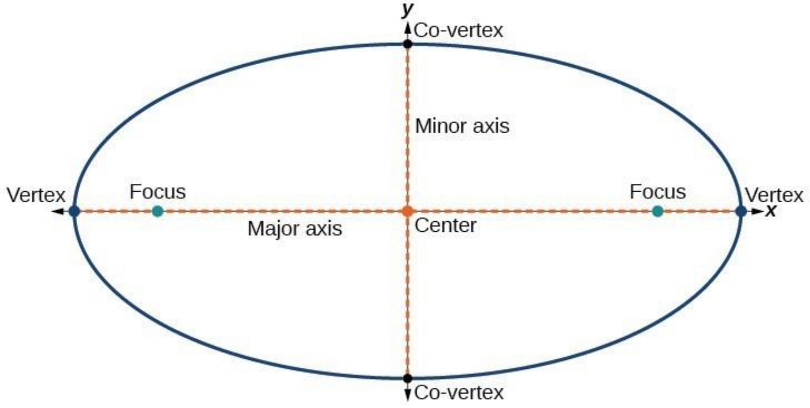
Video



EARTH AS ELLIPSOID

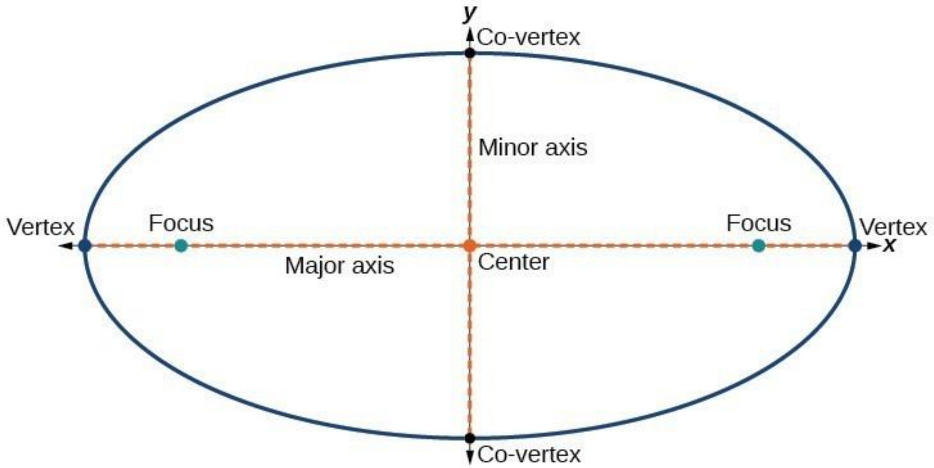
ELLIPSE GEOMETRY

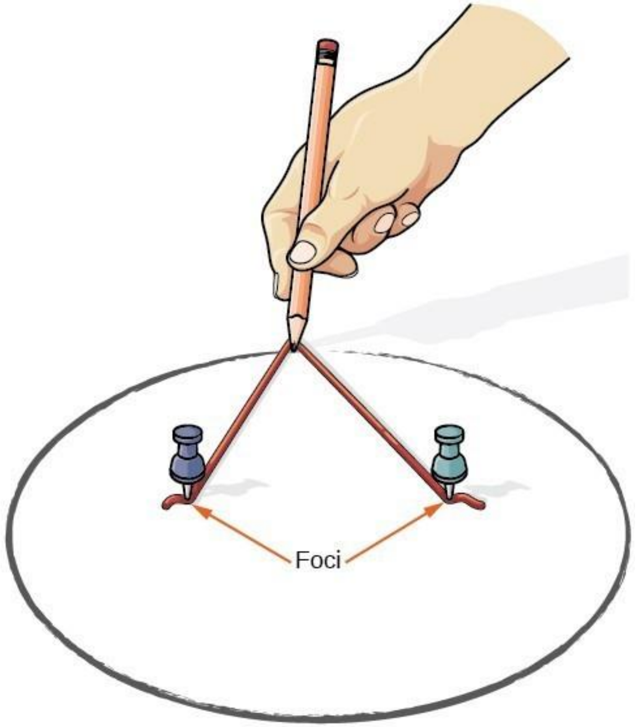
- An **ellipse** is the set of all points (x,y) in a plane such that the sum of their distances from two fixed points is a constant.
- Each fixed point is called a **focus** of the ellipse.
- Every ellipse has two axes of symmetry.
- The longer axis is called the **major axis**, and the shorter axis is called the **minor axis**.
- Each endpoint of the major axis is the **vertex** of the ellipse, and each endpoint of the minor axis is a **co-vertex** of the ellipse.
- The **center of an ellipse** is the midpoint of both the major and minor axes. The axes are perpendicular at the center.
- The foci always lie on the major axis, and the sum of the distances from the foci to any point on the ellipse (the constant sum) is greater than the distance between the foci.

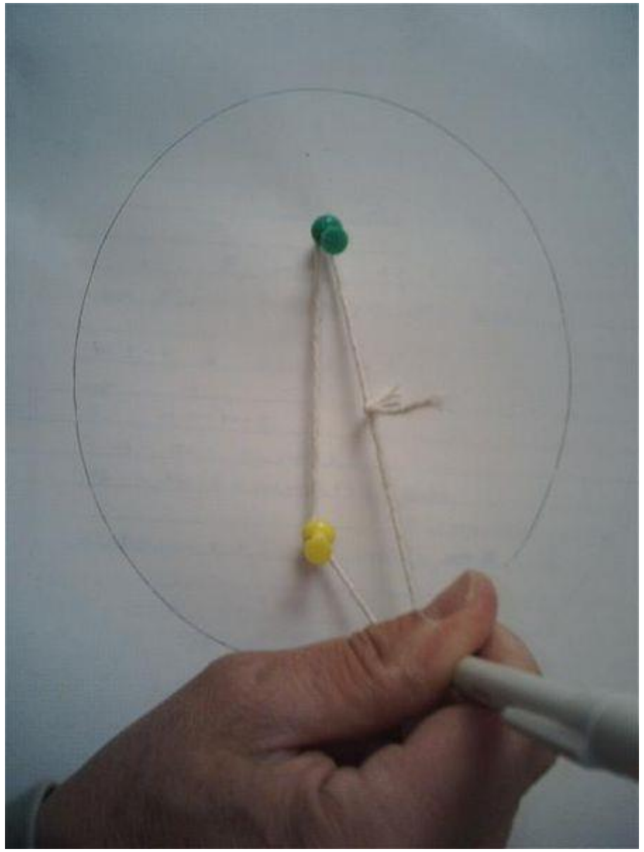


Ellipsoidal Geometry

- **Can draw it?**



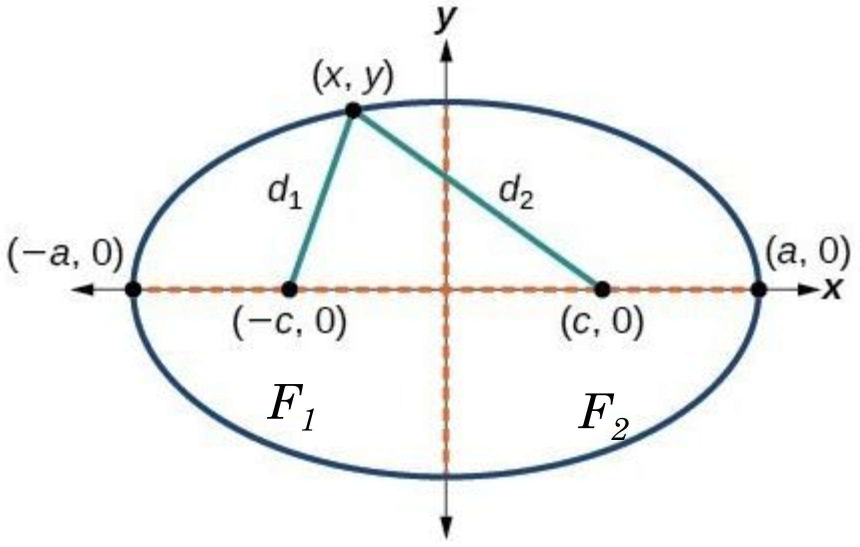




ELLIPSOIDAL GEOMETRY

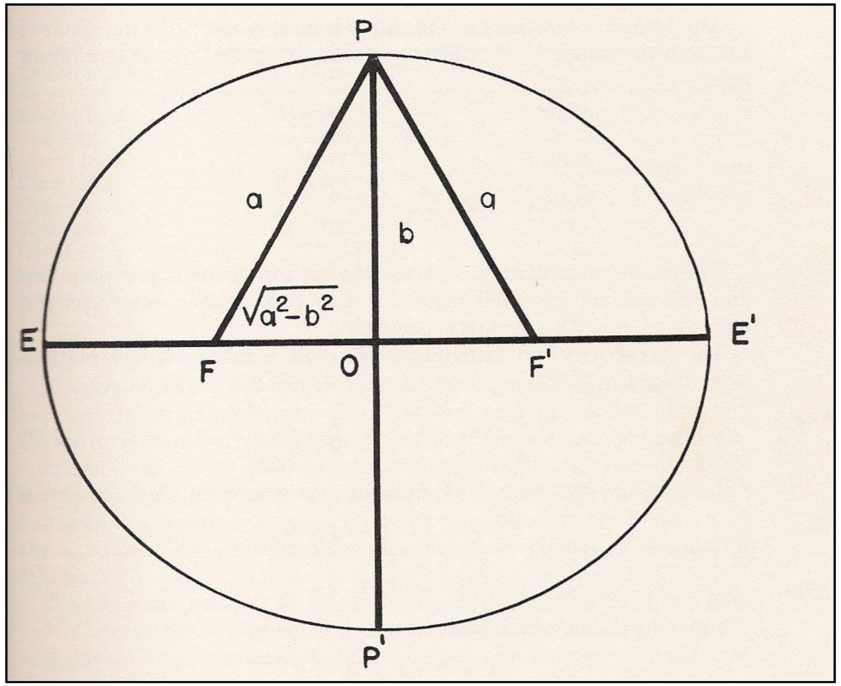
- Introducing a coordinate system (x, y) with origin halfway on the line F_1F_2 and y–axis perpendicular to F_1F_2 , we see that if P is on the x -axis, then that constant is equal to twice the distance from P to the origin; this is the length of the semi-major axis; call it a :

$$\overline{PF_1} + \overline{PF_2} = 2a$$



Ellipsoidal Geometry

- Moving the point, P , to the z -axis, and letting the distance from the origin point to either focal point (F or F') be E , we also find that
- $E = \sqrt{a^2 - b^2}$
- E is called the linear eccentricity of the ellipse (and of the ellipsoid), the distance from the origin point to either focal point.



Equation of Ellipse and Ellipsoid

- The derivation is beyond the scope of this course, but the equation for an ellipse centered at the origin with its major axis on the X-axis:

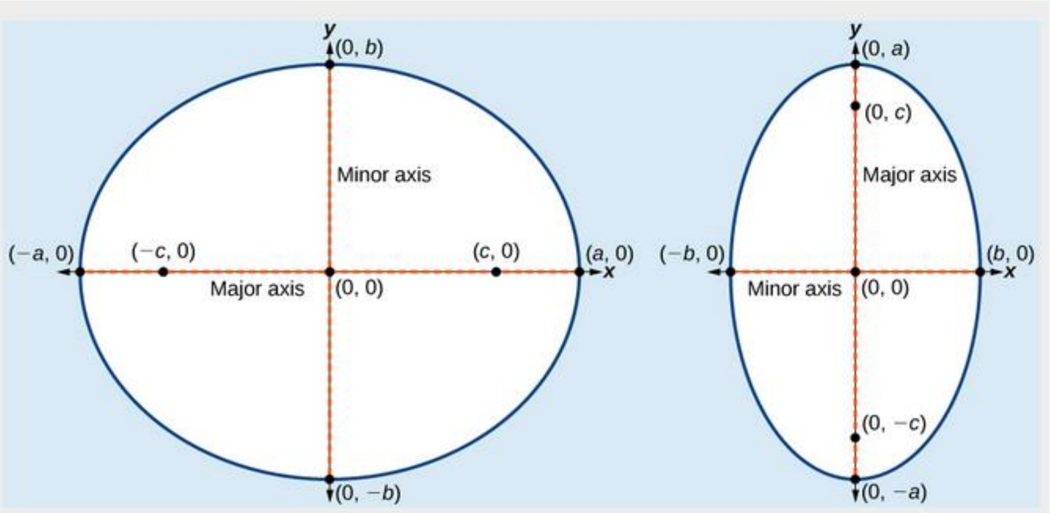
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

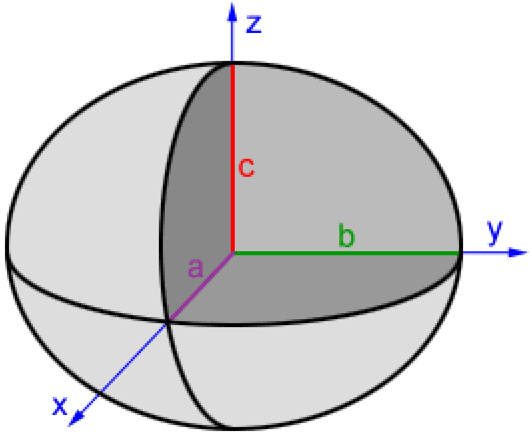
- While for an ellipse centered at the origin with its major axis on the Y-axis, it becomes:-

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

- In 3D, equation of ellipsoid is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$





GEOMETRIC PARAMETERS OF ELLIPSOID

- The ellipsoid is defined by two essential parameters:

(1) a **shape** parameter and,

(2) a **size** (or scale) parameter.

- The **size** parameters of ellipsoid include:

(1) Semi-major axis a

(2) Semi-minor axis b

- The **shape** parameters of ellipsoid include:

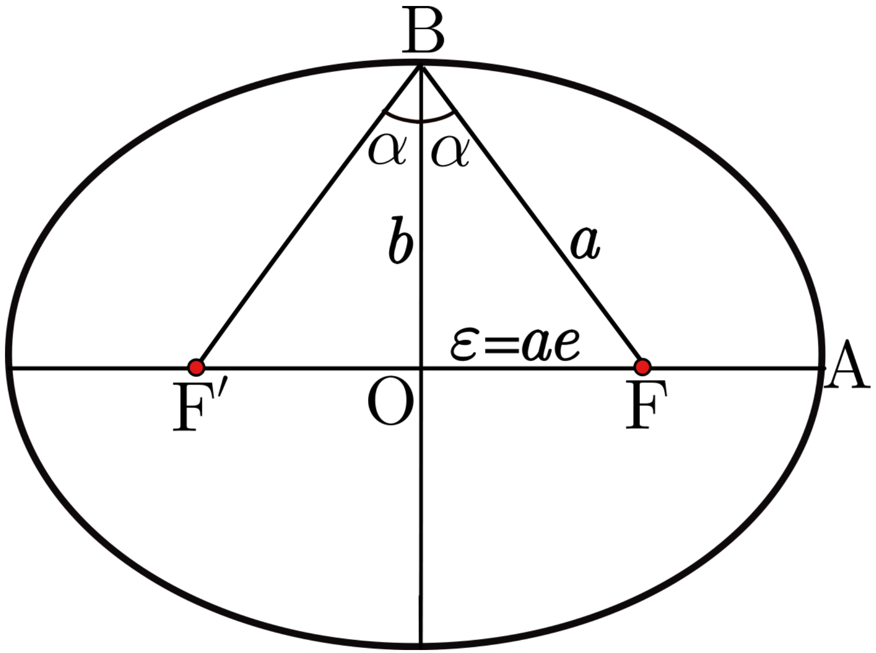
(1) Flattening, $f = \frac{a-b}{a} = 1 - \cos\alpha$

(2) Angular eccentricity, α

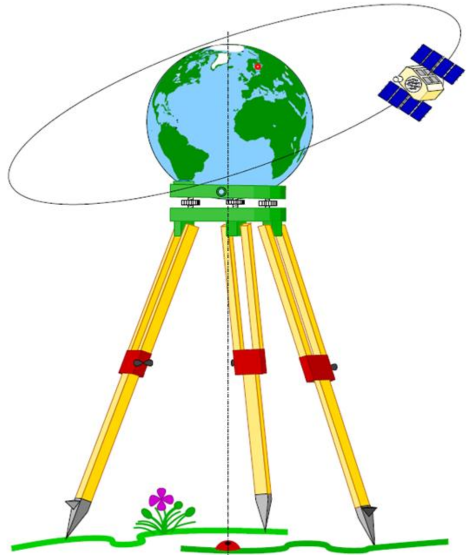
(3) Linear eccentricity, $\varepsilon = \sqrt{a^2 - b^2} = ae$

(4) First eccentricity, $e^2 = \frac{a^2-b^2}{a^2}$

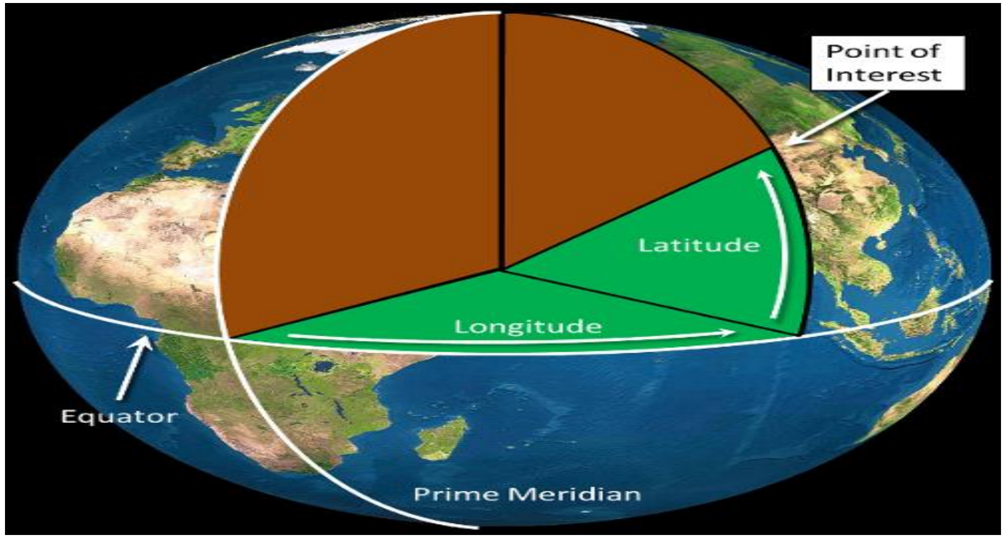
(5) Second eccentricity, $e'^2 = \frac{a^2-b^2}{b^2}$



What are the minimum parameters to define an ellipsoid?

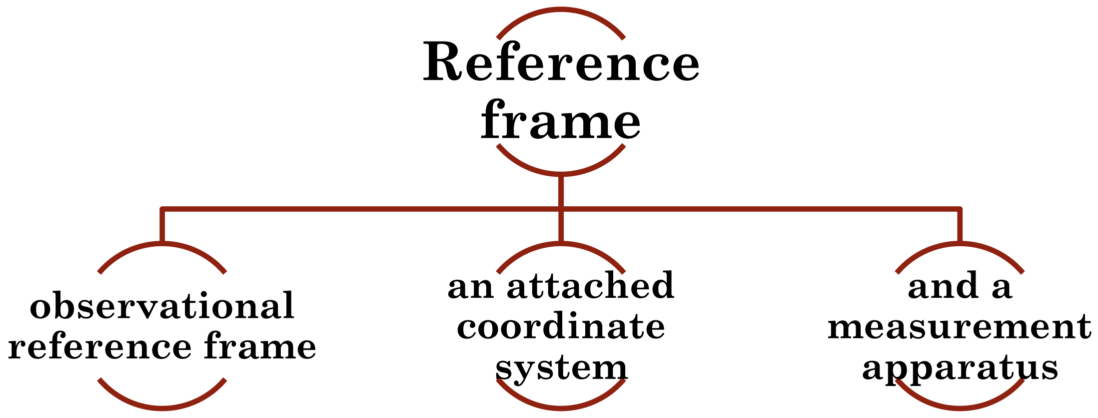


Ellipsoid as Reference Frame



ELLIPSOID AS REFERENCE FRAME

- An **Earth ellipsoid or Earth spheroid** is a mathematical figure approximating the Earth's form (Geoid), used as a reference frame for computations in geodesy, astronomy, and the geosciences.



How is the Earth (Geoid) approximated by ellipsoid?

Ellipsoid as Reference Frame

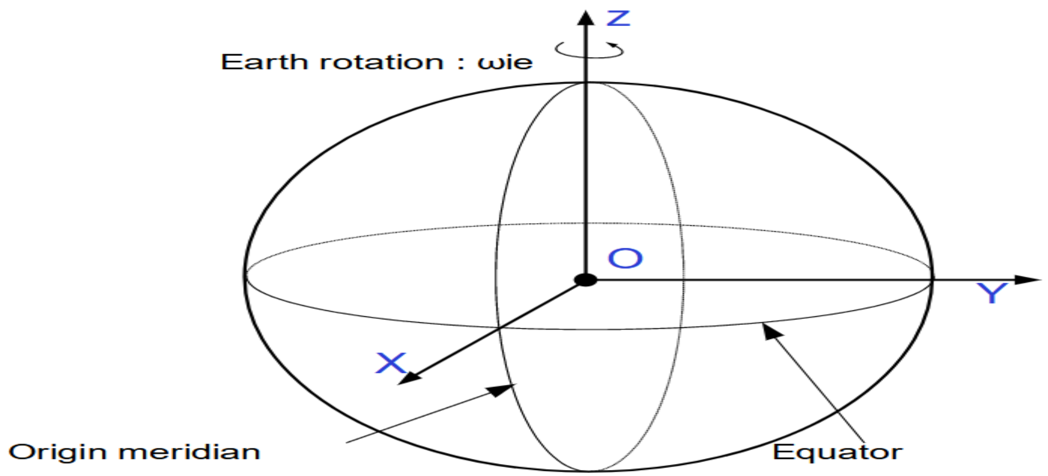
- To answer this question, we need to define the following:

(1) Center of ellipsoid w.r.t earth's center of gravity (CG).

(2) Orientation of ellipsoid axes w.r.t Earth's axes.

(3) Scale between ellipsoid and Earth.

Once these elements have been defined, an ellipsoid could be considered as a **reference frame**.



Ellipsoid as Reference Frame

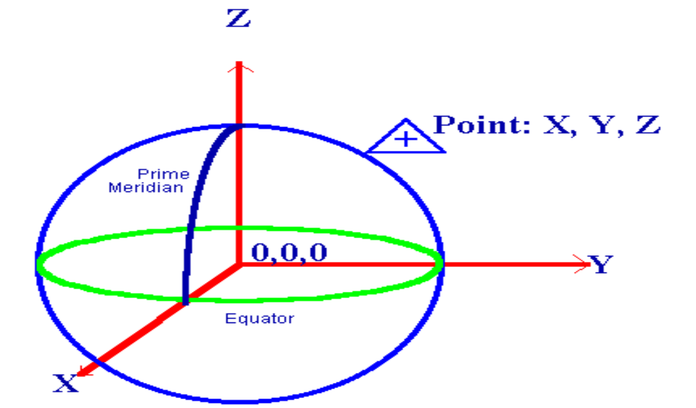
- The simplest datum is an Earth Centered, Earth Fixed cartesian datum(ECEF):–

Origin = center of mass of the Earth's.

– Three right-handed orthogonal axis X, Y, Z.

– Units are meters.

– The Z axis coincides with the Earth's rotation axis



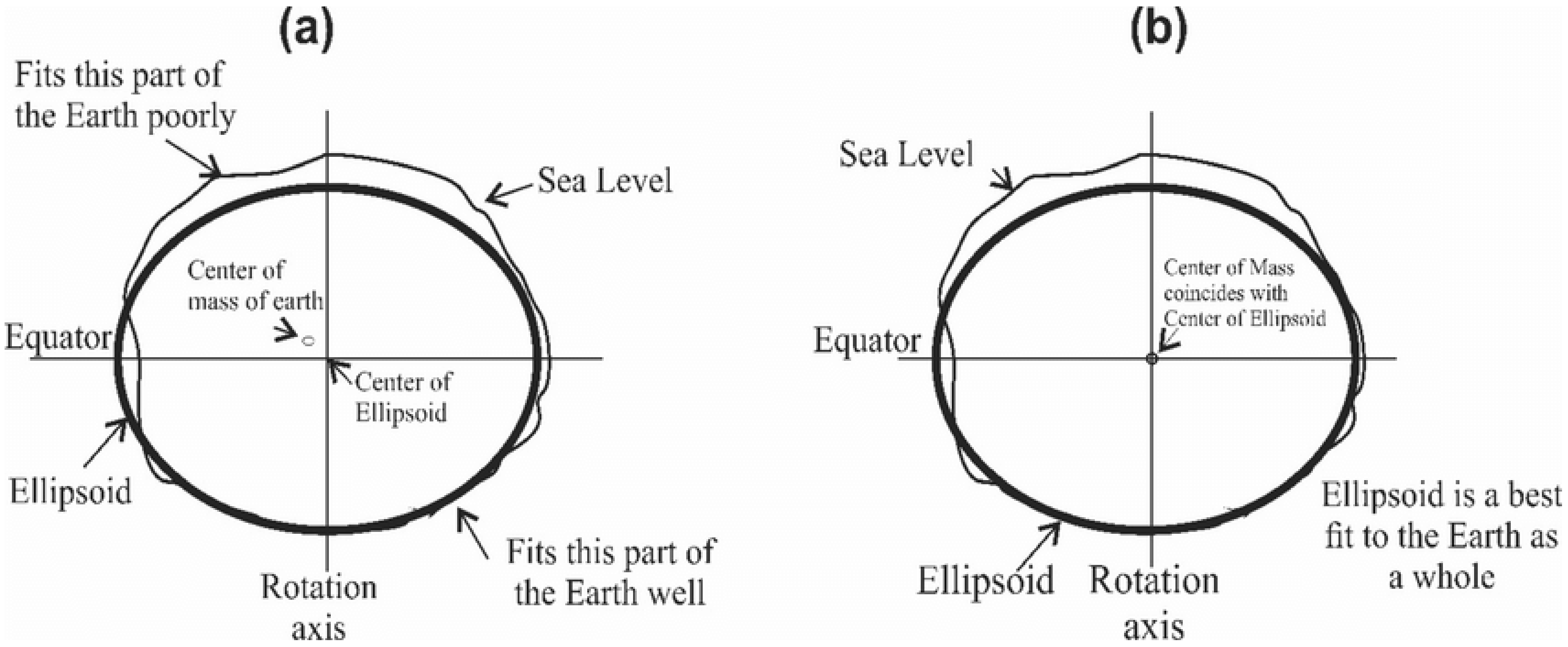
Earth Centered, Earth Fixed X, Y, Z

– The (X,Y) plane coincides with the equatorial plane.

– The (X,Z) plane contains the Earth's rotation axis and the prime meridian.

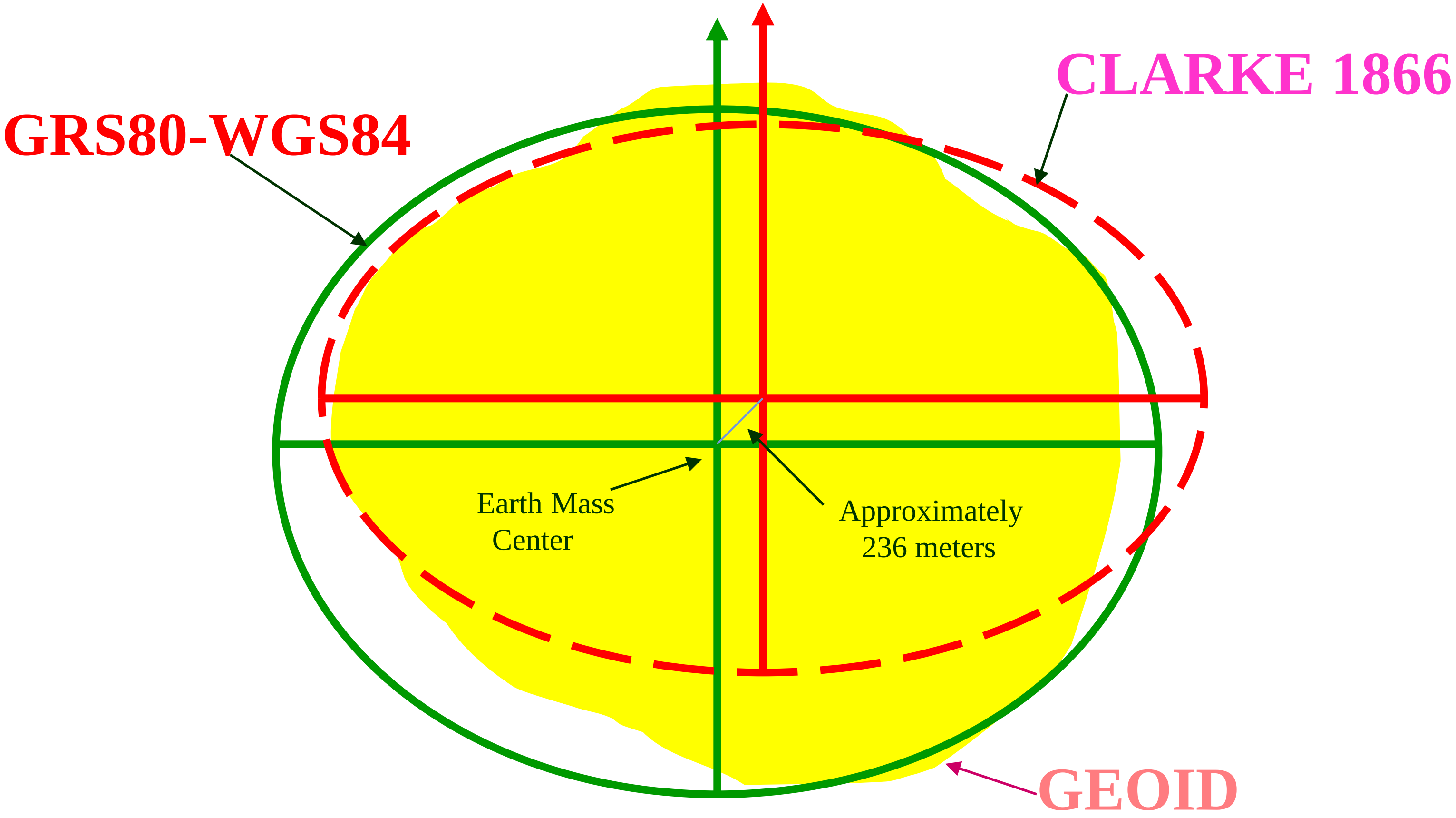
ELLIPSOID AS REFERENCE FRAME (GEOCENTRIC OR TOPOCENTRIC)

- **(a) Topocentric ellipsoid (*Regional/national)***
- **(b) Geocentric ellipsoid (*Global*)**

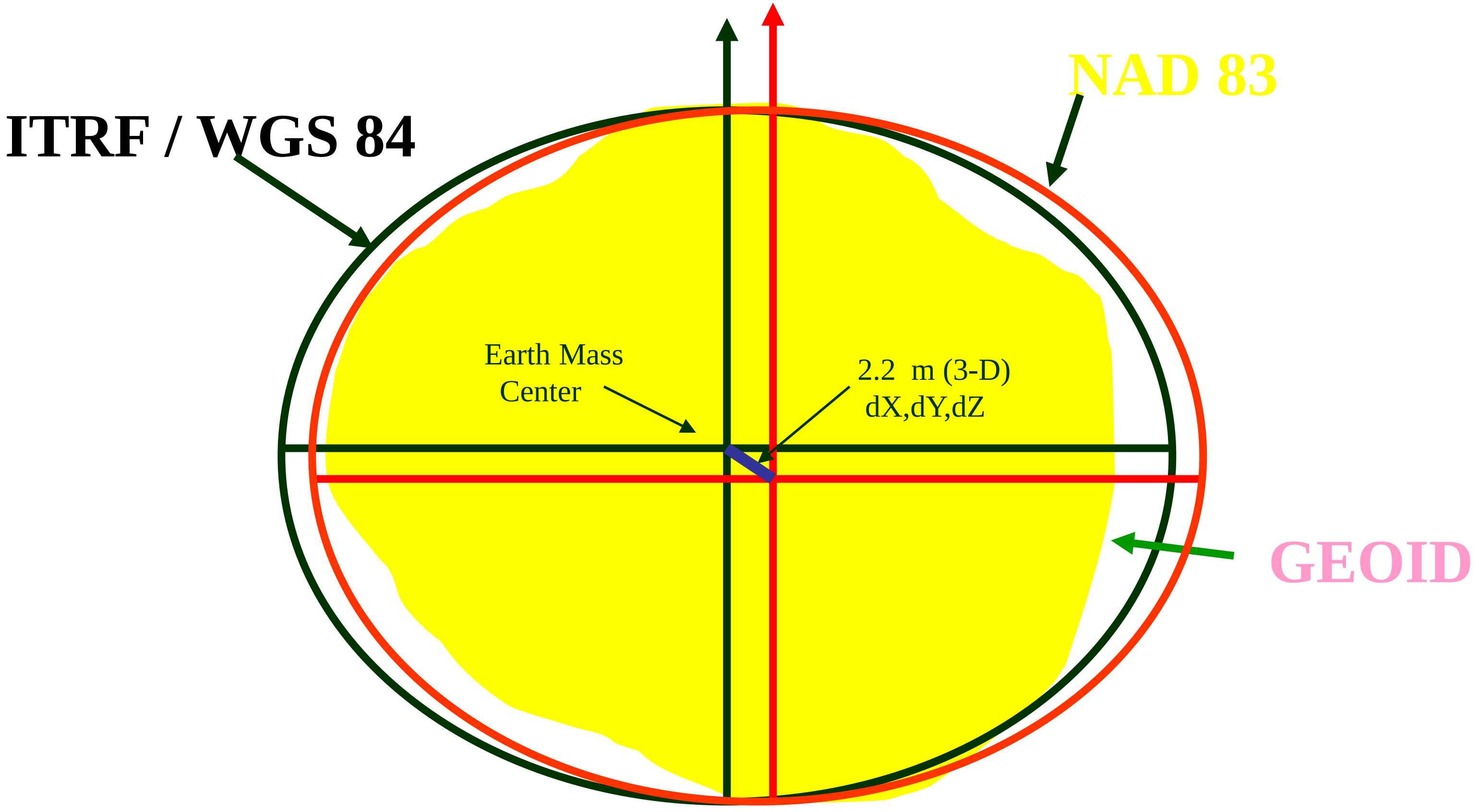


Video

Ellipsoid as Reference Frame (Geocentric or Topocentric)



Ellipsoid as Reference Frame (Geocentric or Topocentric)



Ellipsoid as Reference Frame (List of Ellipsoids)

- **Egypt**: Helmert1906

$$a = 6378.200\ km$$

$$f = \frac{1}{298.3}$$

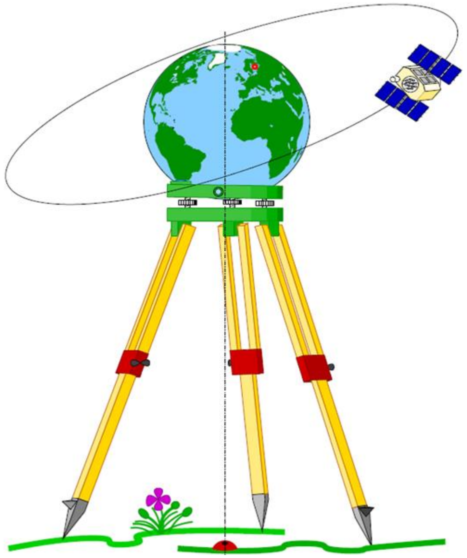
- **Global**: WGS1984

World Geodetic System

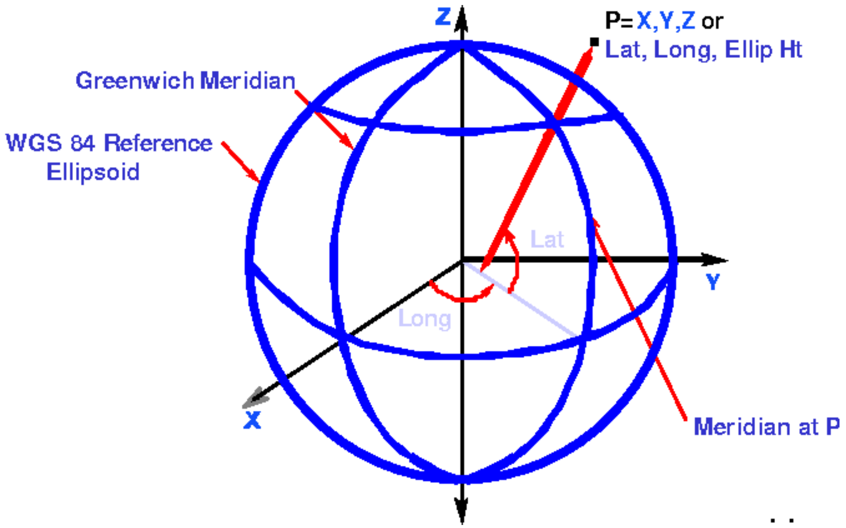
$$a = 6378.137\ km$$

$$f = \frac{1}{298.25642}$$

Reference ellipsoid name	Equatorial radius (m)	Polar radius (m)	Inverse flattening	Where used
Maupertuis (1738)	6,397,300	6,363,806.283	191	France
Plessis (1817)	6,376,523.0	6,355,862.9333	308.64	France
Everest (1830)	6,377,299.365	6,356,098.359	300.80172554	India
Everest 1830 Modified (1967)	6,377,304.063	6,356,103.0390	300.8017	West Malaysia & Singapore
Everest 1830 (1967 Definition)	6,377,298.556	6,356,097.550	300.8017	Brunei & East Malaysia
Airy (1830)	6,377,563.396	6,356,256.909	299.3249646	Britain
Bessel (1841)	6,377,397.155	6,356,078.963	299.1528128	Europe, Japan
Clarke (1866)	6,378,206.4	6,356,583.8	294.9786982	North America
Clarke (1878)	6,378,190	6,356,456	293.4659980	North America
Clarke (1880)	6,378,249.145	6,356,514.870	293.465	France, Africa
Helmert (1906)	6,378,200	6,356,818.17	298.3	Egypt
Hayford (1910)	6,378,388	6,356,911.946	297	USA
International (1924)	6,378,388	6,356,911.946	297	Europe
Krassovsky (1940)	6,378,245	6,356,863.019	298.3	USSR, Russia, Romania
WGS66 (1966)	6,378,145	6,356,759.769	298.25	USA/DoD
Australian National (1966)	6,378,160	6,356,774.719	298.25	Australia
New International (1967)	6,378,157.5	6,356,772.2	298.24961539	
GRS-67 (1967)	6,378,160	6,356,774.516	298.247167427	
South American (1969)	6,378,160	6,356,774.719	298.25	South America
WGS-72 (1972)	6,378,135	6,356,750.52	298.26	USA/DoD
GRS-80 (1979)	6,378,137	6,356,752.3141	298.257222101	Global ITRS[11]
WGS-84 (1984)	6,378,137	6,356,752.3142	298.257223563	Global GPS
IERS (1989)	6,378,136	6,356,751.302	298.257	
IERS (2003)[12]	6,378,136.6	6,356,751.9	298.25642	[11]



Geodetic (Ellipsoidal) Coordinates



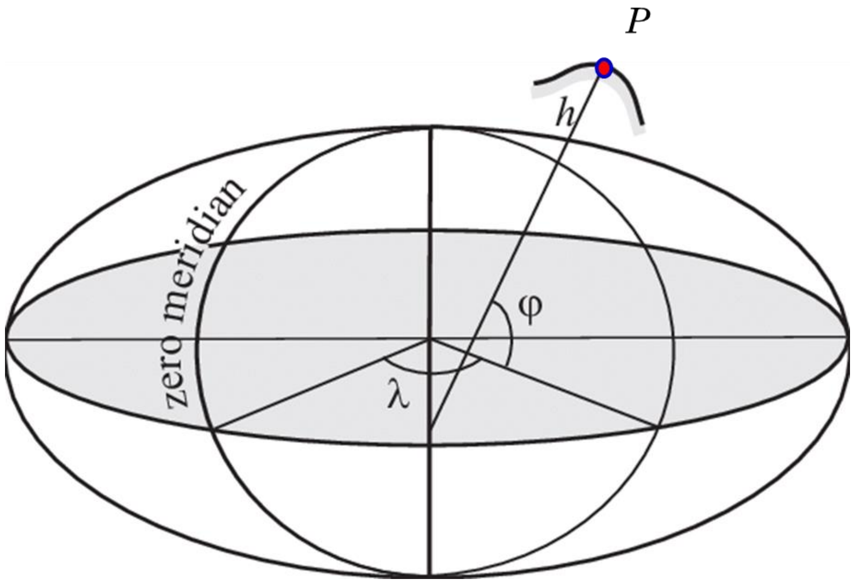
GEODETIC (ELLIPSOIDAL) COORDINATES

- Geodetic coordinates are a type of curvilinear orthogonal coordinate system used in geodesy based on a reference ellipsoid.
- They include:

(1) Geodetic latitude (north/south) φ,

(2) Longitude (east/west) λ, and

(3) Ellipsoidal height h (a.k.a geodetic height).

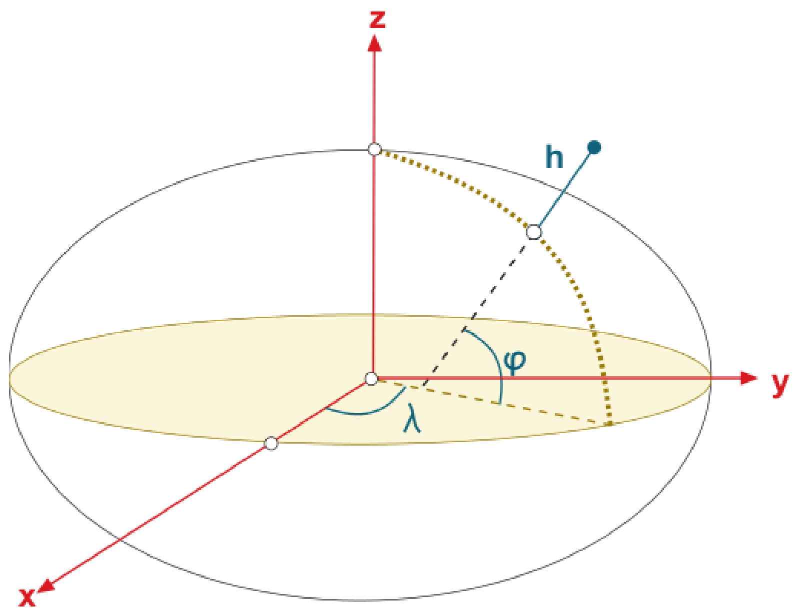


Geodetic (Ellipsoidal) Coordinates

(1) **Geodetic latitude** (north/south) φ: is the angle between the normal to ellipsoid at the observation point and the plane if equator.

(2) **Geodetic Longitude** (east/west) λ : is the *dihedral* angle between the prime meridian (GW) and the meridian of the observation point.

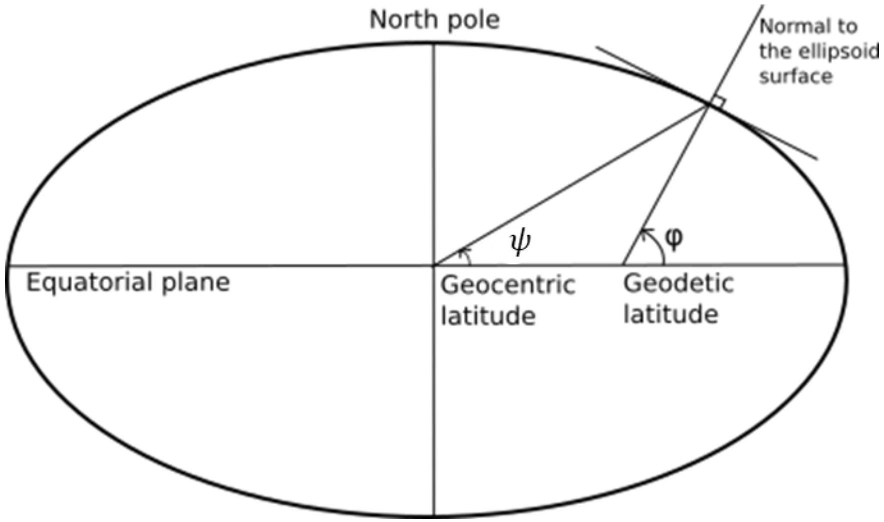
(3) **Ellipsoidal height** h: is the is the height between the ellipsoid and a point on the Earth's surface measured along the normal direction.

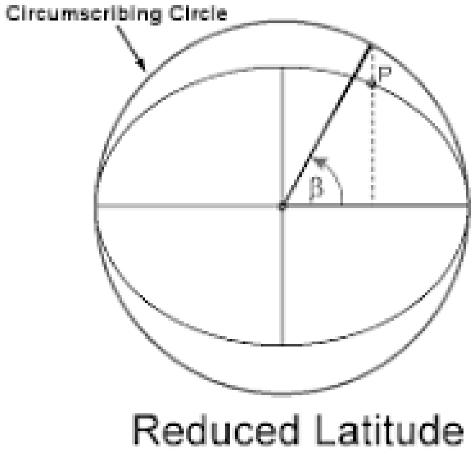


Types Of Latitudes

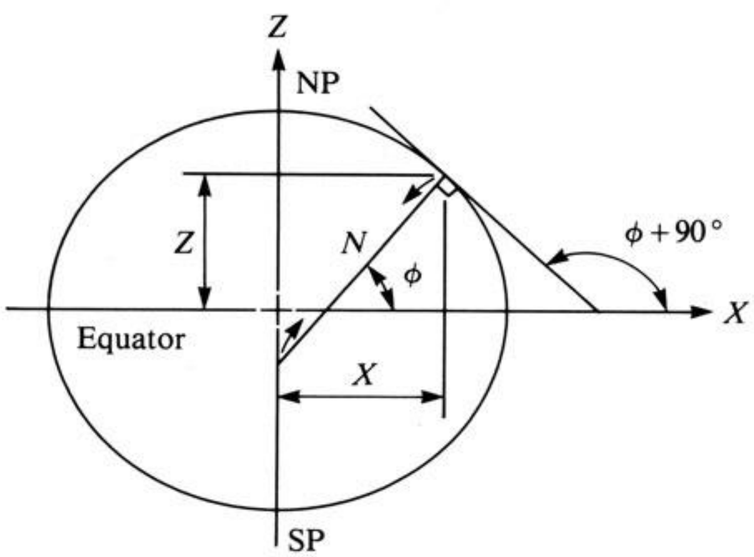
- **In addition to geodetic φ, there are two types of latitudes:**

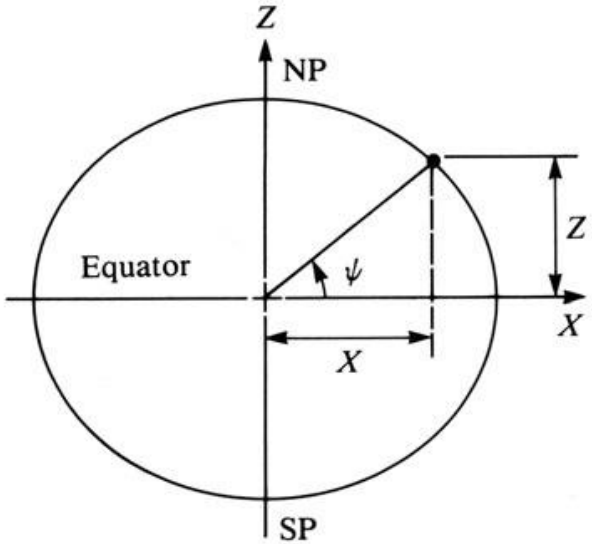
1. **Geocentric Latitude** ψ: the angle between the equatorial plane and a radial line connecting the center of the ellipsoid to a point on the surface.
2. **Reduced Latitude** β: the angle at the center of a sphere which is tangent to the ellipsoid at the equator between the equatorial plane and the radial line to the point.

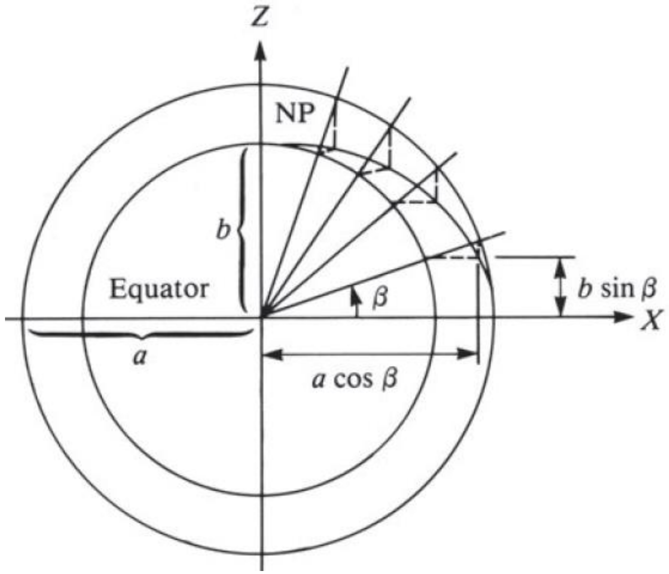




Relation Between Types Of Latitudes







$$X = \frac{a\cos\varphi}{(1 - e^2 sin^2\varphi)^{0.5}}$$

$$\tan\psi = \frac{Z}{X}$$

$$X = a\cos\beta$$
$$Z = b\sin\beta$$

$$Z = \frac{a(1-e^2)\sin\varphi}{(1 - e^2 sin^2\varphi)^{0.5}}$$

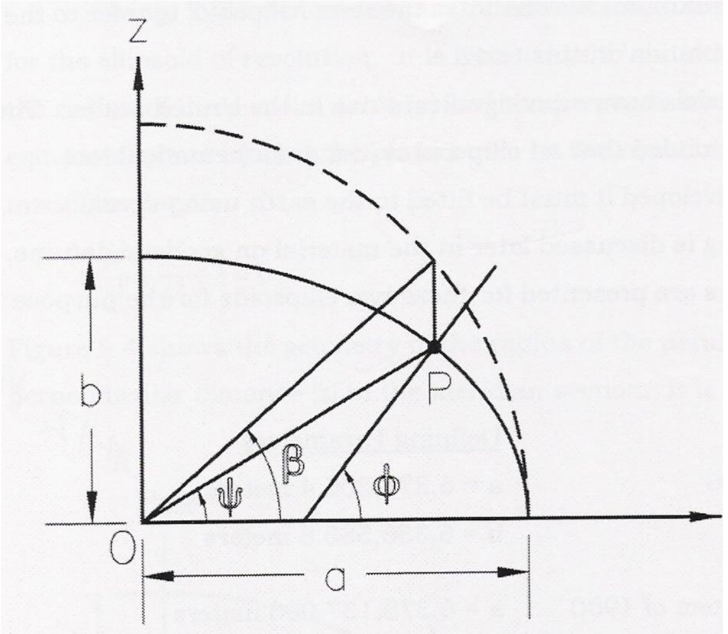
Relation Between Types Of Latitudes

- $\tan\psi = \frac{Z}{X} = \frac{b\sin\beta}{a\cos\beta} = \frac{b}{a}\tan\beta = (1 - e^2)^{0.5}\tan\beta$

- $\tan\psi = \frac{Z}{X} = \frac{\frac{a(1-e^2)\sin\varphi}{(1-e^2\sin^2\varphi)^{0.5}}}{\frac{a\cos\varphi}{(1-e^2\sin^2\varphi)^{0.5}}} = (1 - e^2)\tan\varphi$

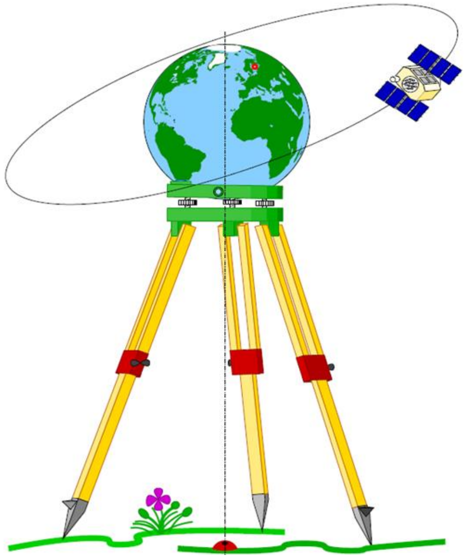
- **Therefore,**

$$\tan\psi = (1 - e^2)^{0.5}\tan\beta = (1 - e^2)\tan\varphi$$

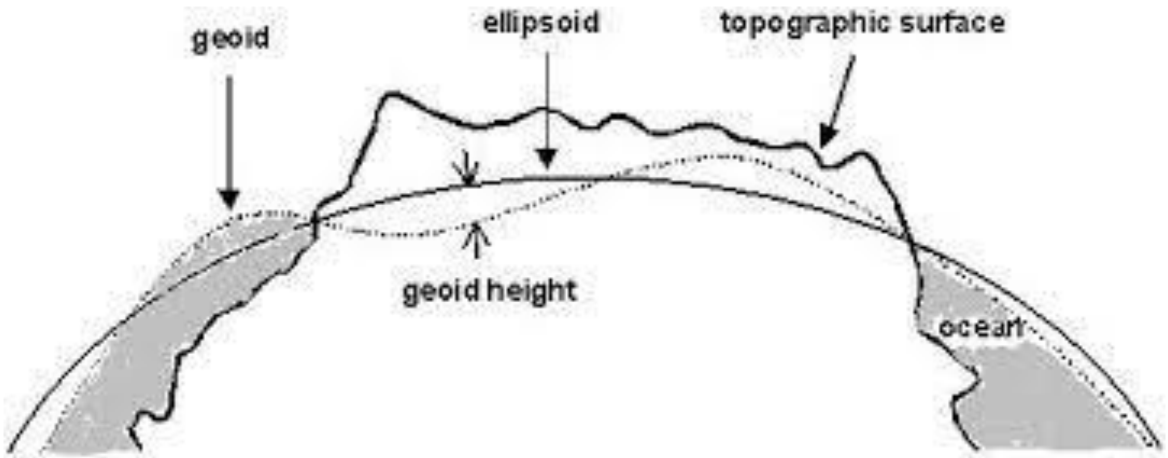


The three latitudes

How could you interpret this relationship?

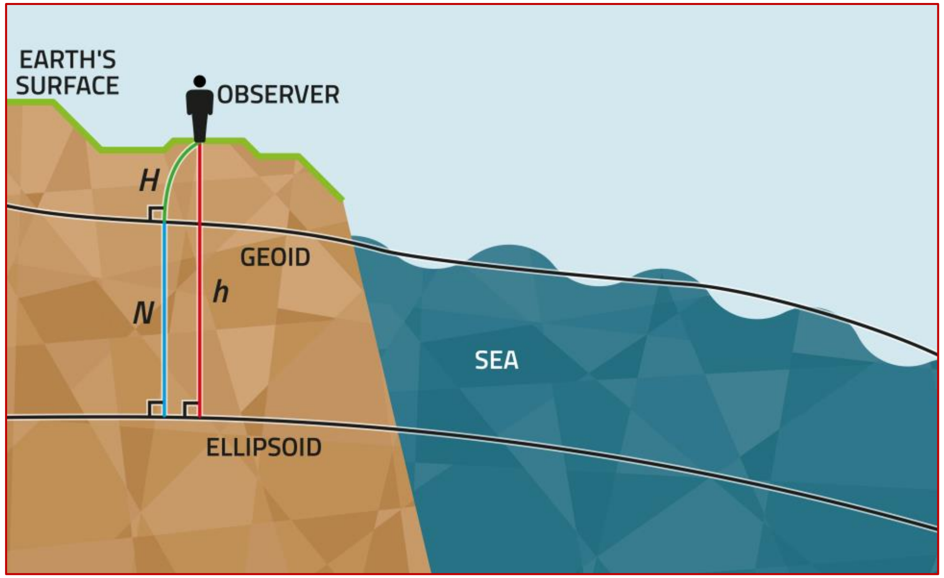


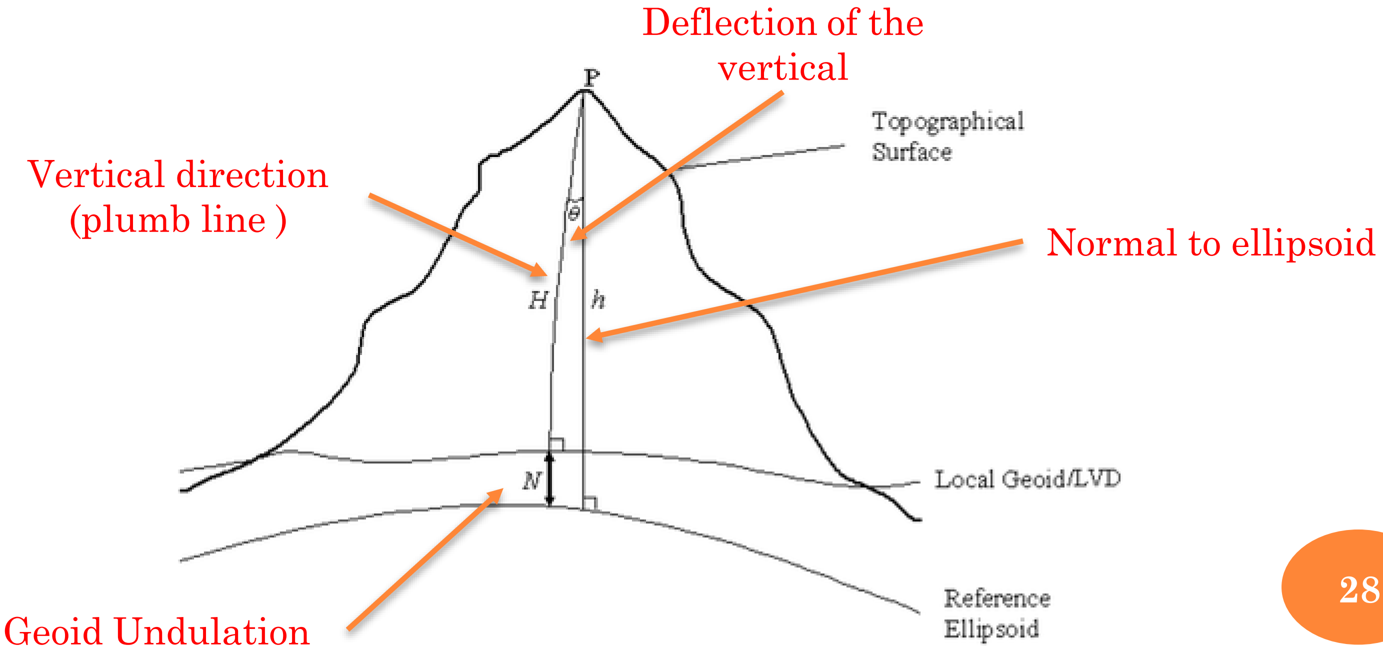
Relation Between Topography, Ellipsoid and Geoid



RELATION BETWEEN TOPOGRAPHY, ELLIPSOID AND GEOID

- Topographic surface represents the earth's surface.
- Geoid coincides with the MSL.
- Computations are made on the surface of ellipsoid.





Relation Between Topography, Ellipsoid and Geoid

- Based on the figure, the relationship is characterized by:

(1) One **linear** component : which is represented by the geoidal undulation $\boldsymbol{N}$. The undulation can be positive or negative depending the position of ellipsoid w.r.t geoid.

(2) One **angular** component: which is represented by the angle of deflection of the vertical θ.

$$N = h - H,$$

H: orthometric height obtained from spirit leveling.

h: ellipsoidal height.

- **Accordingly, think about .....**

(1) Under what circumstances can an ellipsoid and a geoid align in a parallel manner?

(2) What would happen if N = 0 or $\theta = 0$? Also, if both = 0?

(3) ....

RELATION BETWEEN TOPOGRAPHY, ELLIPSOID AND GEOID

- **Deflection of the vertical θ has two components:**

North-south component ξ, positive toward the north:-

$\xi = \Phi - \varphi$

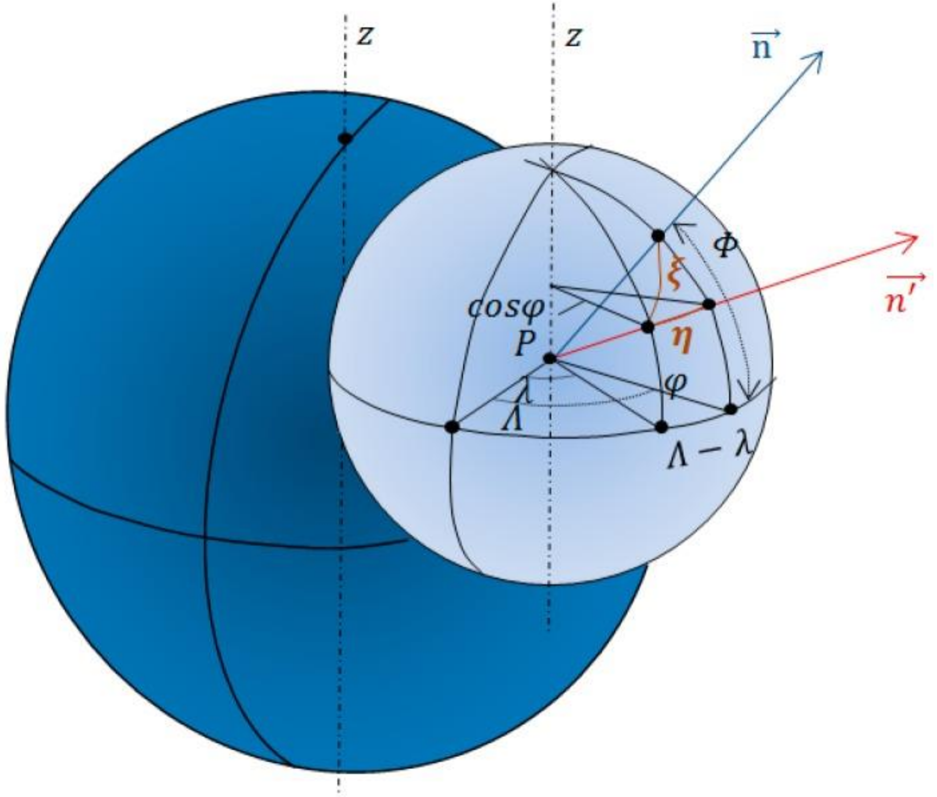
East-west component η, positive toward the east:-

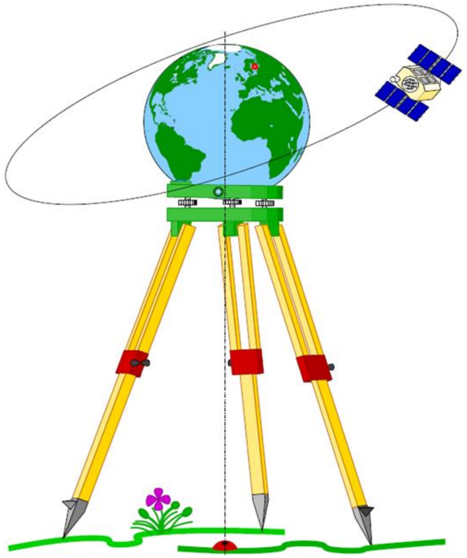
$\eta = (\Lambda - \lambda)\cos\varphi$

Such that:

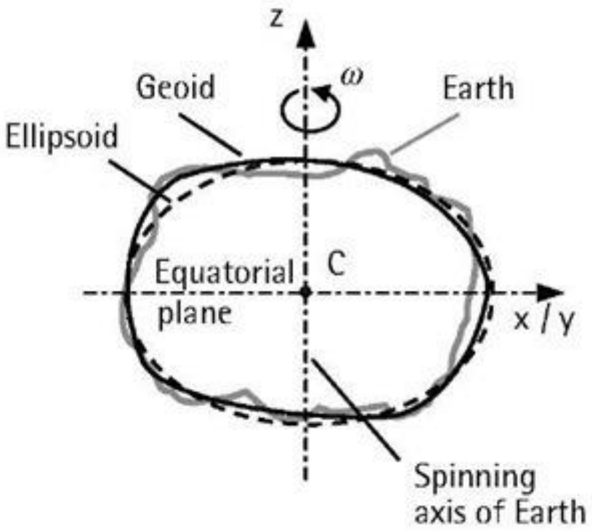
Φ: Astronomic Latitude.

Λ: Astronomic Longitude.





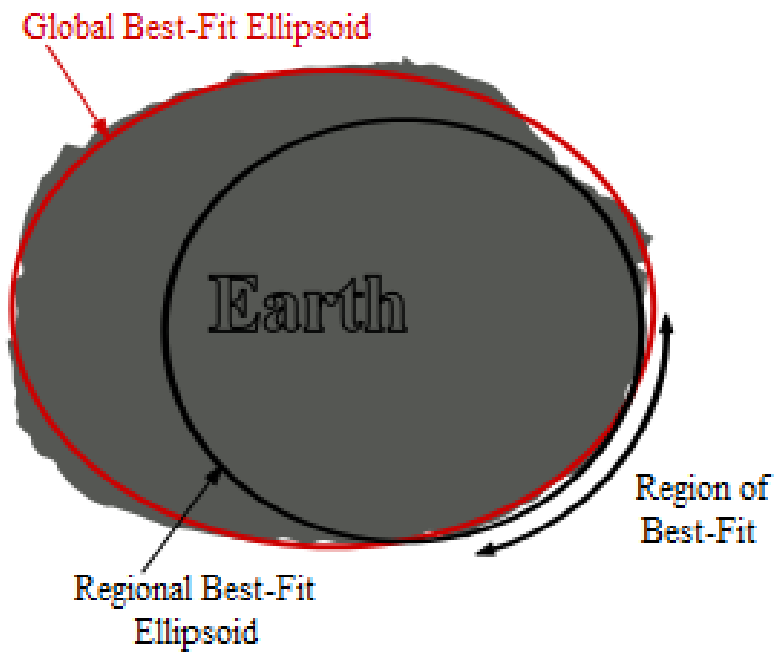
How could Ellipsoid Be Best-Fit to Geoid?



Best Fitting Ellipsoid

- An ellipsoid satisfying the condition that the deviations between the geoid and ellipsoid (in a global sense) are minimized.

  <u>*Computed by least squares methods.*</u>



- Distance N between the geoid and best-fitting ellipsoid is called geoidal *undulation* and can be computed from: $N \approx h - H$.

LET'S SUMMARIZE

We need to test "Kahoot"

NEXT TUESDAY

Lecture 3 - Curves on Ellipsoid

Be Prepared

THANK YOU

End of Presentation